INTERFERENCE BETWEEN PLANE SURFACES IN SUPERSONIC GAS FLOW

(INTERFERENTSIIA MEZHDU PLOSKIMI POVERKHNOSTIAMI V SVERKHZVUKOVOM POTOKE GAZA)

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A solution of the problem of interference between two parallel-situated wings in supersonic gas flow was obtained by a method of Volterra [1] with the help of a transformation of Fal'kovich [2]. A similar transformation has also been used by Fridlender [3] for an analysis of the flow past a cruciform wing.

In that paper the possibility of using this method for the investigation of the flow within a rectangular parallelepiped is pointed out. In contrast to this, in the present paper the solution of this problem is obtained through its reduction to the problem of flow between two parallel wings.

1. Over a system of two slender wings, situated one over the other, flows a supersonic stream of ideal gas. We direct the x-axis of a rectangular coordinate system x, y, z along the direction of flow of the stream, and choose the axes y and z such that with zero angle of attack α, the wing surfaces differ little from the planes $z = 0$ and $z = h$. We designate by S^- the region in the plane $z = 0$ in which the lower wing gives rise to a disturbance, and by S^+ the disturbed region of the upper wing in the plane $z = h$ (see Fig.1). We will consider the velocity potential of the perturbed flow to satisfy the wave equation

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial y^2} - \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{1.1}$$

Here we have taken the Mach number at infinity to be $\sqrt{2}$. For the other values of M the Prandtl-Glauert transformation is used.

We consider first the case where the leading edge of one wing does not influence the leading edge of the other. Let us take, in the space between wings, the point $p\ (x_p, y_p, z_p)$. The characteristic cone with vertex at this point, going in the direction of decreasing values of x, cuts out on the surfaces S^+ and S^- corresponding regions $S^+(x_p, y_p, z_p)$ and $S^-(x_p, y_p, z_p)$. Application to the point p of the Volterra formula [1] yields

$$\Phi(x_p, y_p, z_p) = \frac{1}{2\pi}\frac{\partial}{\partial x_p}\iint\limits_{S^+(p)+S^-(p)}\left(U(p)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(p)}{\partial n}\right)dx\,dy \tag{1.2}$$

Here

$$U(p) = \lg\left[\frac{x_p - x + \sqrt{(x_p - x)^2 - (z_p - z)^2 - (y_p - y)^2}}{\sqrt{(z_p - z)^2 + (y_p - y)^2}}\right]$$

and n is the direction of the outward normal to the surfaces S^+ and S^-.

In the derivation of Equation (1.2) it was taken into account that on the characteristic surfaces Σ_1, which divide the regions of perturbed and unperturbed flows, the potential Φ is zero.

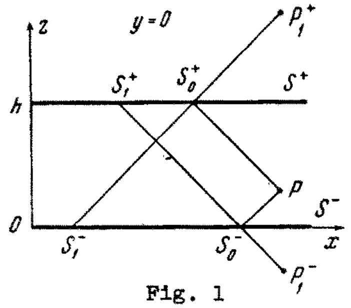

Fig. 1

Let us take now the point q (x_q, y_q, z_q), lying outside of the space between the wings. The characteristic cone with vertex at this point cuts out in the regions S^+ and S^- certain portions $S^+(q)$ and $S^-(q)$, which, together with the sides of the characteristic cone and the surface Σ, form a closed region. The application of Green's formula [1] to the functions Φ and $U(q)$ in this region yields

$$\iint\limits_{S^+(q)+S^-(q)} \left[U(q)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(q)}{\partial n}\right] dx\,dy = 0 \qquad (1.3)$$

We choose the point q symmetrical to the point p relative to the plane $z = h$, and designate it by p_1^+. In consequence of the symmetry of the points p and p_1^+ we have on the plane $z = h$

$$U(p_1^+) = U(p), \quad \partial U(p_1^+)/\partial n = -\partial U(p)/\partial n, \quad S^+(p_1^+) = S^+(p) \qquad (1.4)$$

From Equation (1.3), using (1.4), we obtain

$$\iint\limits_{S^+(p)} \Phi\frac{\partial U(p)}{\partial n} dx\,dy = -\iint\limits_{S^+(p)} U(p)\frac{\partial\Phi}{\partial n} dx\,dy - \iint\limits_{S^-(p_1^+)} \left[U(p_1^+)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(p_1^+)}{\partial n}\right] dx\,dy \qquad (1.5)$$

If we now apply an analogous transformation for the point $p_1^-(x_p, -y_p, z_p)$, symmetric to the point p relative to the plane $z = 0$, we have as a result

$$\iint\limits_{S^-(p)} \Phi\frac{\partial U(p)}{\partial n} dx\,dy = -\iint\limits_{S^-(p)} U(p)\frac{\partial\Phi}{\partial n} dx\,dy - \iint\limits_{S^+(p_1^-)} \left[U(p_1^-)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(p_1^-)}{\partial n}\right] dx\,dy \qquad (1.6)$$

Substituting (1.5) and (1.6) in (1.2), we get

$$\Phi(p) = \frac{1}{2\pi}\frac{\partial}{\partial x_p}\left\{2\iint\limits_{S^+(p)} U(p)\frac{\partial\Phi}{\partial n} dx\,dy + 2\iint\limits_{S^-(p)} U(p)\frac{\partial\Phi}{\partial n} dx\,dy + \right. \qquad (1.7)$$

$$\left. + \iint\limits_{S^-(p_1^+)} \left[U(p_1^+)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(p_1^+)}{\partial n}\right]\partial x\,\partial y + \iint\limits_{S^+(p_1^-)} \left[U(p_1^-)\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial U(p_1^-)}{\partial n}\right] dx\,dy\right\}$$

It is evident that $S^+(p_1^-) < S^+(p)$ and $S^-(p_1^+) < S^-(p)$. It might turn out that the regions $S^+(p_1^-) = 0$ and $S^-(p_1^+) = 0$; if, however, this is not the case, then transformation (1.3) can be applied next to the region $S^+(p_1^-)$ or $S^-(p_1^+)$, taking point p_2^+ or p_2^- to be symmetric, with respect to point p_1^- or p_1^+, relative to the plane $z = h$ or $z = 0$. This process is continued until in the right part of Formula (1.7) all integrals containing the function Φ vanish, and we obtain

$$\Phi(p) = \frac{1}{\pi}\frac{\partial}{\partial x_p}\sum_{i=0}\left[\iint\limits_{S^+(p_i^-)} U(p_i^-)\frac{\partial\Phi}{\partial n} dx\,dy + \iint\limits_{S^-(p_i^+)} U(p_i^+)\frac{\partial\Phi}{\partial n} dx\,dy\right] \qquad (1.8)$$

where

$$z_{p_i^-} = -z_{p_{i-1}^+}, \quad z_{p_i^+} = -z_{p_{i-1}^-} + 2h$$

The series in (1.8) terminates when the points p_i^+, p_i^- leave the region of influence of the wings.

Performing the differentiation in Equation (1.8), we find

$$\Phi(p) = \sum_{i=0} [\Phi^+(p_i^-) + \Phi^-(p_i^+)] \tag{1.9}$$

where

$$\Phi^+(p_i^-) = \frac{1}{\pi} \iint\limits_{S^+(p_i^-)} \frac{\partial\Phi / \partial n \, dx\, dy}{\sqrt{(x_p - x)^2 - (y_p - y)^2 - (z_{p_i^-} - h)^2}}$$

$$\Phi^-(p_i^+) = \frac{1}{\pi} \iint\limits_{S^-(p_i^+)} \frac{\partial\Phi / \partial n \, dx\, dy}{\sqrt{(x_p - x)^2 - (y_p - y)^2 - z_{p_i^+}^2}} \tag{1.10}$$

From (1.10) it is seen that the function $\Phi^+(p_{p_i^-})$ is none other than the value of the potential, at the point p_i^-, for flow over the isolated upper wing, and $\Phi^-(p_i^+)$ is the potential, at the point p_i^+, for flow over the isolated lower wing. Thus the potential for flow between two parallel-situated wings is expressed through the potentials for flow over these wings when isolated.

If the leading edges of the wings are supersonic, then Formula (1.10) immediately gives the solution to the problem of flow between two surfaces, since the derivative $d\Phi/dn$ is known in this case from the boundary condition. In the case of a subsonic leading edge the determination of the potential for flow over an isolated wing also does not give rise to difficulties of principle [4].

2. We assume now that the supersonic leading edge of the lower wing lies in the region of influence of the upper wing. Designate by Φ_0 the potential for flow over an isolated upper wing.

We think now of the lower wing as being continued up to the intersection with the characteristic surface passing through the leading edge of the upper wing, and set $\partial\Phi/\partial n = \partial\Phi_0/\partial n$ on the new surface. We would then solve the problem with boundary conditions on the upper and new, fictitious, lower wings. Obviously this new problem is equivalent to the old, and its solution is given by Formula (1.9).

3. Let us consider supersonic flow within a hollow rectangular parallelepiped, whose boundaries differ little from the planes $z = 0$, $z = h$, $y = 0$, $y = b$. We designate the projections of the boundary surfaces onto these planes by S^+, S^-, Q^+ and Q^-, respectively. The linearized boundary conditions will have the form

$$\partial\Phi / \partial z = F^+(x, y) \quad \text{on } S^+, \qquad \partial\Phi / \partial z = F^-(x, y) \quad \text{on } S^- \tag{3.1}$$

$$\partial\Phi / \partial y = L^+(x, z) \quad \text{on } Q^+, \qquad \partial\Phi / \partial y = L^-(x, z) \quad \text{on } Q^- \tag{3.2}$$

and on the characteristic surfaces passing through the leading edges of the boundaries, $\Phi = 0$.

We will seek a solution to the posed problem in the form of a sum $\Phi = \Phi_0 + \Phi_1$. Function Φ_0 must satisfy conditions (3.1) and, besides these,

$$\partial\Phi_0 / \partial y = 0 \quad \text{on } Q^+ \text{ and } Q^- \tag{3.3}$$

The potential Φ_1, in contrast, must satisfy relations (3.2), and on the surfaces S^+ and S^- it is required that $\partial\Phi_1/\partial z = 0$. The sum of functions thus selected obviously solves the posed problem. We will find the function Φ_0.

Let us consider an auxiliary problem. Let the equations of the leading edges of the surfaces S^+ and S^- be given, respectively, in the form

$$x = x^+(y), \qquad x = x^-(y) \qquad (0 \leqslant y \leqslant b) \tag{3.4}$$

We form the surface $S^{\cdot +}$, infinite in the direction of the y-axis, with leading edge given by Equation $x = x^{\cdot +}(y)$, where

$$x^{\cdot +}(-y) = x^{\cdot +}(y), \qquad x^{\cdot +}(y \pm 2bk) = x^{\cdot +}(y) \qquad (k \text{ is the integer})$$

$$x^{\cdot +}(y) = x^{+}(y), \qquad \text{if } b \geqslant y \geqslant 0$$

that is, the region $S^{\cdot +}$ coincides with the region S^{+} in the interval $[0, b]$, and each plane $y = \pm kb$ is a plane of symmetry for this surface. We also form, in a similar way, the region $S^{\cdot -}$ on the basis of the region S^{-}.

We define now, on the region $S^{\cdot +}$, the function $F^{\cdot +}$ by the equalities

$$F^{\cdot +}(x, -y) = F^{\cdot +}(x, y), \qquad F^{\cdot +}(x, y \pm 2kb) = F^{\cdot +}(x, y)$$

$$F^{\cdot +}(x, y) = F^{+}(x, y), \qquad \text{if } b \geqslant y \geqslant 0$$

The function $F^{\cdot +}(x,y)$ will thus be symmetric with respect to each plane $y = \pm kb$. On the surface $S^{\cdot -}$ we construct, in a similar way, the function $F^{\cdot -}(x,y)$, taking as basis the function $F^{-}(x,y)$.

We will consider now the problem of the wave equation with the following boundary conditions:

$$\frac{\partial \Phi^{\cdot}}{\partial z} = F^{\cdot +}(x, y) \quad \text{on } S^{\cdot +}, \qquad \frac{\partial \Phi^{\cdot}}{\partial z} = F^{\cdot -}(x, y) \quad \text{on } S^{\cdot +} \qquad (3.5)$$

Since boundary conditions (3.5) are symmetric with respect to the planes $y = 0$ and y b , then the solution also must by symmetric with respect to these planes, and the condition $d\Phi^{\cdot}/dy = 0$ for $y = 0$ and $y = b$ must automatically be satisfied. In addition, the boundary conditions (3.5) agree in the interval $[0, b]$ with conditions (3.1) by construction. The function $\Phi^{\cdot}$ satisfies all the conditions established for the function Φ_0, and it follows that they are identically equal in the interval $[0, b]$. However, solutions to the problem for Φ were obtained in Section 1. The determination of the potential Φ_1 is mathematically indistinguishable from the problem just considered. Thus the potential for supersonic flow within a hollow rectangular parallelepiped is represented in the form of a quadrature.

Obviously, a solution can be obtained in the same way if the papallelepiped has only three faces.

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